Breakdown of the Entropy/Area Relationship for NUT-charged Spacetimes

Dumitru Astefanesei,[1][*] Robert B. Mann,[2][†] Eugen Radu[3][‡]

[1]Department of Physics, McGill University Montréal, Québec H3A 2T8, Canada

[1,2]Perimeter Institute for Theoretical Physics, Ontario N2J 2W9, Canada

[2]Department of Physics, University of Waterloo Waterloo, Ontario N2L 3G1, Canada

[3]Department of Mathematical Physics, National University of Ireland, Maynooth, Ireland

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Abstract

We demonstrate that $(D+1)$-dimensional, locally asymptotically anti-deSitter spacetimes with nonzero NUT charge generically do not respect the usual relationship between area and entropy. This result does not depend on either the existence of closed timelike curves nor on the removal of Misner string singularities, but instead is a consequence of the first law of thermodynamics.

The relationship between thermodynamic entropy and the area of an event horizon has simultaneously been one of the more robust and intriguing concepts in gravitational physics [1]. It holds for both black hole and cosmological event horizons, and has natural extensions to their dilatonic variants [2]. Recovering the entropy/area relationship is generally regarded as a litmus test that all quantum theories of gravity must pass [3].

In a very basic sense gravitational entropy can be regarded as arising from the Gibbs-Duhem relation applied to the path-integral formulation of quantum gravity [4], which in the semiclassical limit yields a relationship between gravitational entropy and other relevant thermodynamic quantities such as mass and angular momentum. In asymptotically de Sitter (dS) or anti de Sitter (AdS) spacetimes such quantities can be computed intrinsically, without reference to a background, by employing the AdS/CFT-inspired counterterm approach [5]. Whenever it is not possible to foliate the Euclidean section of a given (stationary) spacetime by a family of surfaces of constant time, gravitational entropy will emerge [6]. This situation can occur in $(D+1)$-dimensions if the topology of the Euclidean section is not trivial – specifically when the (Euclidean) timelike Killing

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[*]E-mail: `dastefanesei@perimeterinstitute.ca`

[†]E-mail: `mann@avatar.uwaterloo.ca`

[‡]E-mail: `radu@thphys.may.ie`

vector $\xi = \partial/\partial\tau$ that generates the $U(1)$ isometry group has a fixed point set of even co-dimension. If this co-dimension is $(D-1)$ then the usual relationship between area and entropy holds.

It has recently become apparent that spacetimes containing a NUT charge furnish situations in which the co-dimension is smaller than this, generalizing the relationship between area and entropy [6, 7]. Such spacetimes can be constructed by taking the fibration of a circle over a space of constant curvature. If this curvature is positive (e.g. a product of p $-S^2$'s and/or CP^2's) [8, 9] the orbits of the $U(1)$ isometry group develop singularities, of dimension $(D-2p)$ in the orbit space, and of dimension $(D-2p+1)$ in the Euclidean section. These singularities are the gravitational analogues of Dirac string singularities and are referred to as Misner strings [10]. If the curvature is non-positive, then Misner strings are not present [11].

Here we demonstrate that – regardless of whether or not there are Misner strings present – the entropy/area relationship does not hold for asymptotically AdS spacetimes containing NUT charge. This result in stands in contrast to previous work, in which periodicity β of the foliation for constant positive curvature was adjusted so that singularities from neither the bolts nor the Misner strings are present, which in turn was taken to imply that the gravitational entropy is no longer proportional to the area of the bolts [6, 11, 12]. We find instead that breakdown of entropy/area proportionality is a consequence of the first law of thermodynamics, which imposes the constraint that the periodicity of the foliation (interpreted as the inverse temperature) is proportional to the NUT charge. If the fibration is over a (compact) space of zero or negative curvature there are no Misner strings, and there are no constraints on the constant of proportionality. However if the foliation is over a space of constant positive curvature there will be Misner string singularities, which can be removed by ensuring that constant of proportionality is an inverse integer. Our results imply that the relationship between entropy and area is much less generic than previously supposed, with Misner strings doing nothing more than quantizing the constant of proportionality relating NUT charge to inverse temperature.

Since the boundary metric of a $(D+1)$-dimensional Taub-Nut-AdS solution provides a D-dimensional generalization of the known Gödel-type spacetimes[1], our solutions provide a useful tool with which to investigate the physics in spacetimes containing closed timelike curves (CTCs) via the AdS/CFT correspondence. Previous investigations have indicated that such spacetimes are generally believed to be unstable with respect to quantum fluctuations. However these studies were of spacetimes with confined causality violation, i.e. with CTCs confined to some region, with at least one region CTC-free [13]. Cauchy horizons separate the latter regions from the former. This is not the case for the Gödel spacetime, where causality violation is not a result of the evolution of certain initial data, but rather has existed "forever".

The $(D+1)$-dimensional vacuum Taub-NUT solutions (with $D = 2p+1$), constructed by taking a $U(1)$ fibration over $(M^2)^{\otimes p}$ have metrics of the form

$$ds^2 = \frac{dr^2}{F(r)} + G(r)(d\theta_i^2 + f_k^2(\theta_i)d\varphi_i^2) - F(r)(dt + 4nf_k^2(\frac{\theta_i}{2})d\varphi_i)^2, \tag{1}$$

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[1]These solutions can be embedded in supergravity theories (of 10 or 11 dimensions) and may constitute consistent backgrounds of string theory [14].

with i summed from 1 to p, and $f_k(\theta)$ given by

$$
f_k(\theta) = \begin{cases} \sin \theta, & \text{for } k = 1 \\ \theta, & \text{for } k = 0 \\ \sinh \theta, & \text{for } k = -1, \end{cases}
$$

where we take the space M^2 of constant curvature to be positive (a 2-sphere ($k = 1$)), zero (a plane ($k = 0$)) or negative (a pseudohyperboloid ($k = -1$)). By solving the Einstein equation in $(D+1)$ dimensions with cosmological constant $\Lambda = -D(D-1)/2\ell^2$, we find $G(r) = r^2 + n^2$, while the general form for $F(r)$ is found to be

$$
F(r) = \frac{r}{(r^2 + n^2)^{(D-1)/2}} \left(\int^r \left[k \frac{(s^2 + n^2)^{(D-1)/2}}{s^2} + \frac{D}{\ell^2} \frac{(s^2 + n^2)^{(D+1)/2}}{s^2} \right] ds - 2M \right),
$$

where the parameter M is an integration constant related to the spacetime mass. In four dimensions we recover the known form $F = (k(r^2 - n^2) - 2Mr + (r^4 + 6n^2r^2 - 3n^4)/\ell^2)/(r^2 + n^2)$ [12].

For generic values of the parameters, the spacetime will have singularities at $\theta_i = \pi$ only in the $k = 1$ case. These are the aforementioned Misner string singularities; they do not appear in the $k = 0, -1$ cases. For any k, the Riemann tensor and its derivatives remain finite in all parallelly propagated orthonormal frames, and no curvature scalars diverge. The only remaining singularities are (for $k = 1$) quasiregular singularities, which are the end points of incomplete and inextendible geodesics that spiral infinitely around a topologically closed spatial dimension. World lines of observers approaching these points come to an end in a finite proper time [15].

To compute the relevant thermodynamic quantities, we analytically continue in the time coordinate and the NUT charge $t \to i\tau, \quad n \to iN$ in order to obtain a solution on the Euclidean section. For any value of k, there will be conical singularities at the roots r_+ of the function $F(r)$ unless the Euclidean time coordinate has the periodicity

$$
\beta = \left| \frac{4\pi}{F'(r_+)} \right|, \tag{2}
$$

which affords a thermodynamic interpretation of β as the inverse temperature [4]. Furthermore, the mass parameter M must be restricted such that the fixed point set of the Killing vector ∂_τ is a regular at the radial position $r = r_+$ (defined by setting $F(r_+) = 0$). Two kinds of solutions emerge: "bolts", with $r_+ = r_b > N$, for which the fixed point set is of dimension $(D-1)$, and "nuts" ($r_+ = N$), for which the fixed point set is less than this maximal value. Note that each kind of spacetime has nonzero NUT charge N.

The metrics (1) extremize the gravitational action

$$
I_G = -\frac{1}{16\pi G} \int_{\mathcal{M}} d^{D+1}x \sqrt{-g} \, (R - 2\Lambda) - \frac{1}{8\pi G} \int_{\partial \mathcal{M}} d^D x \sqrt{-\gamma} \Theta. \tag{3}
$$

The first term in this relation is the bulk action over the $(D+1)$-dimensional manifold $\mathcal{M}$ with metric g and the second term is a surface term necessary to ensure that the Euler-Lagrange

variation is well-defined, γ being the induced metric of the boundary. However, even at tree-level, the action (3) contains divergences that arise from integrating over the infinite volume of spacetime. These divergences can be removed by adding additional boundary terms that are geometric invariants of the induced boundary metric, leading to a finite total action [5]. In the context of AdS/CFT this action corresponds to the partition function of the CFT. An algorithmic prescription exists [16] for computing this additional boundary action, and the result is

$$
\begin{aligned}
I_{\text{ct}} \ = \ & \frac{1}{8\pi G} \int d^D x \sqrt{-\gamma} \left\{ -\frac{D-1}{\ell} - \frac{\ell\Theta\,(D-3)}{2(D-2)}\mathsf{R} - \frac{\ell^3\Theta\,(D-5)}{2(D-2)^2(D-4)} \left(\mathsf{R}_{ab}\mathsf{R}^{ab} - \frac{D}{4(D-1)}\mathsf{R}^2 \right) \right. \\
& + \frac{\ell^5\Theta\,(D-7)}{(D-2)^3(D-4)(D-6)} \left(\frac{3D+2}{4(D-1)}\mathsf{R}\mathsf{R}^{ab}\mathsf{R}_{ab} - \frac{D(D+2)}{16(D-1)^2}\mathsf{R}^3 \right. \\
& \left. \left. -2\mathsf{R}^{ab}\mathsf{R}^{cd}\mathsf{R}_{acbd} - \frac{D}{4(D-1)}\nabla_a\mathsf{R}\nabla^a\mathsf{R} + \nabla^c\mathsf{R}^{ab}\nabla_c\mathsf{R}_{ab} \right) + ... \right\},
\end{aligned}
\tag{4}
$$

where R and R^{ab} are the curvature and the Ricci tensor associated with the induced metric γ. The series truncates for any fixed dimension, with new terms entering at every new odd value of D, as denoted by the step-function ($\Theta\,(x) = 1$ provided $x \geq 0$, and vanishes otherwise). We emphasize that the use and validity of the counterterm method does not depend upon the validity of AdS/CFT [16].

The total action is $I = I_G + I_{\text{ct}}$, and from this one can compute

$$
T_{ab} = \frac{2}{\sqrt{-h}} \frac{\delta I}{\delta \gamma^{ab}},
\tag{5}
$$

which is a divergence-free boundary stress tensor, whose explicit expression for $D \leq 8$ is given in ref. [17]. Provided the boundary geometry has an isometry generated by a Killing vector ξ^μ, a conserved charge

$$
\mathfrak{Q}_\xi = \oint_\Sigma d^{D-1}S^a \ \xi^b T_{ab},
$$

can be associated with a closed surface Σ (with normal n^a). If $\xi = \partial/\partial t$ then $\mathfrak{Q}$ is the conserved mass/energy $\mathfrak{M}$.

Gravitational thermodynamics is then formulated via the Euclidean path integral

$$
Z = \int D\,[g]\,D\,[\Psi]\,e^{-I[g,\Psi]} \simeq e^{-I},
$$

where one integrates over all metrics and matter fields between some given initial and final Euclidean hypersurfaces, taking τ to have some period β, determined from eq. (2) by requiring the Euclidean section be free of conical singularities. Semiclassically the total action is evaluated from the classical solution to the field equations, yielding an expression for the entropy

$$
S = \beta(\mathfrak{M} - \mu_i\mathfrak{C}_i) - I,
\tag{6}
$$

upon application of the Gibbs-Duhem relation to the partition function [7] (with chemical potentials $\mathfrak{C}_i$ and conserved charges μ_i). The specific heat C is computed from $C = -\beta\partial S/\partial\beta$. The first law of thermodynamics is then

$$
dS = \beta(d\mathfrak{M} - \mu_i d\mathfrak{C}_i).
\tag{7}
$$

For $k = 1$, an expression for the mass, temperature and finite gravitational action in arbitrary dimensions was derived in ref. [18]. These arguments are straightforwardly extended to any value of k, yielding

$$I_{k,D+1} = \frac{V\beta}{8\pi G}\left[M - \frac{D}{l^2}\sum_{i=0}^{(D-1)/2}\binom{\frac{(D-1)}{2}}{i}(-1)^i N^{2i}\frac{r_+^{(D-2i)}}{(D-2i)}\right],\tag{8}$$

where $V = \int\prod_{i=1}^p f_k(\theta_i)d\theta_i d\varphi_i$ is the total area of the sector (θ_i, φ_i) (with $V = (4\pi)^{(D-1)/2}$ for $k = 1$). The conserved mass-energy is

$$\mathfrak{M} = \frac{(D-1)VM}{8\pi G},\tag{9}$$

where for all nut/bolt solutions the value of M can be computed in terms of r_+ by the condition $F(r = r_+) = 0$

$$\begin{aligned}M_{k,D+1} &= \frac{1}{2}\int^{r_+}\frac{ds}{s^2}\left((k(s^2 - N^2)^{\frac{D-1}{2}} + \frac{D}{\ell^2}(s^2 - N^2)^{\frac{(D+1)}{2}}\right) \\ &= \frac{1}{2}\left(\frac{(r_+^2 - N^2)^{(D+1)/2}}{l^2 r_+} + (k - \frac{N^2(D+1)}{l^2})\int^{r_+}ds\frac{(s^2 - N^2)^{(D+1)/2}}{s^2}\right).\end{aligned}\tag{10}$$

Bolt solutions exist for every value of k. The condition $F(r_b > N) = 0$ implies that the τ coordinate has periodicity

$$\beta = \frac{4\pi r_b \ell^2}{k\ell^2 + D(r_b^2 - N^2)},\tag{11}$$

with $r_+ = r_b$ in eq. (10). The action and entropy of the bolt solutions are given by the general expressions

$$\begin{aligned}I_{k,D+1}^b &= \frac{V\beta}{16\pi G\ell^2}\left(-\frac{(r_b^2 - N^2)^{(D+1)/2}}{r_b} + \left(k\ell^2 - (D-1)N^2\right)\int^{r_b}ds\frac{(s^2 - N^2)^{(D-1)/2}}{s^2}\right),\tag{12}\\ S_{k,D+1}^b &= \frac{V\beta}{16\pi G\ell^2}\left(\frac{D(r_b^2 - N^2)^{(D+1)/2}}{r_b} + \left((D-2)k\ell^2 - D(D-1)N^2\right)\int^{r_b}ds\frac{(s^2 - N^2)^{(D-1)/2}}{s^2}\right),\end{aligned}$$

where the latter quantity is not proportional to the area $V(r_b^2 - N^2)^{(D-1)/2}$ of the bolt.

Upon insertion of the expressions (10)-(12) (viewed as functions of (r_b, N)) into the relation (7), we find that for a generic $r_b > N$, the coefficients of dr_b and dN are nonzero, and so the first law of thermodynamics fails to be satisfied. This suggests that the bolt radius is not independent of the NUT charge and/or the cosmological constant. We can avoid this contradiction by supposing that $r_b = r_b(N)$ and demanding that these coefficients vanish. We then find that the first law of thermodynamics (7) holds if and only if

$$r_{b\pm} = \frac{\sigma\ell^2 \pm \sqrt{\sigma^2\ell^4 + D(D+1)^2 N^2\left(DN^2 - k\ell^2\right)}}{D(D+1)N},\tag{13}$$

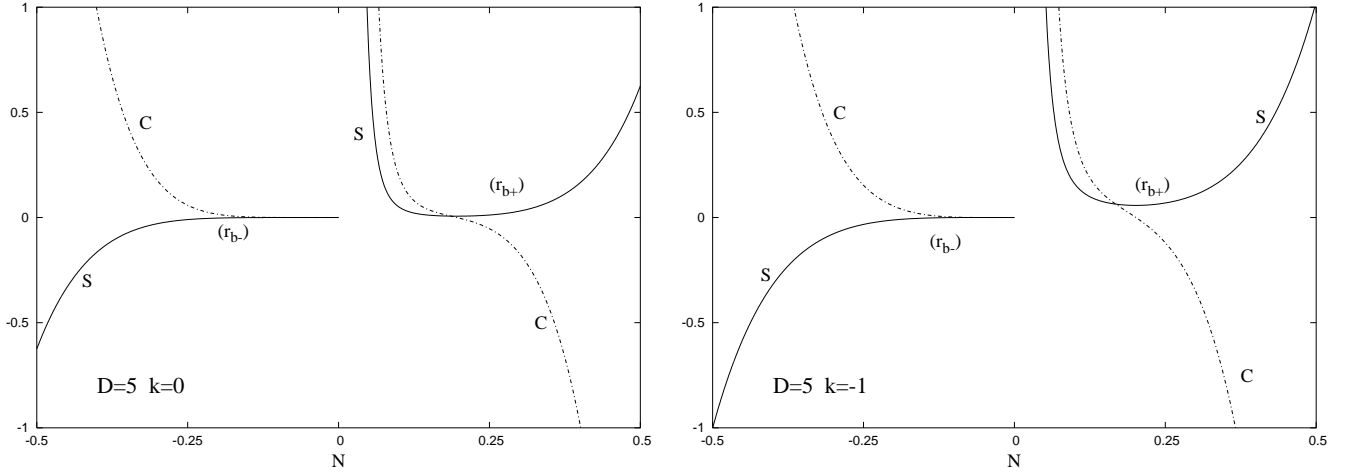**Figure 1.** The entropy and specific heat of $k = 0, -1$ solutions are represented as a function of N for bolt solutions in six dimensions.

where σ is an arbitrary parameter. Insertion of this relationship into eq. (2) implies $\beta = 2\pi (D+1) N/\sigma$.

Remarkably, we find that a consistent thermodynamics forces a relation between the NUT charge and the bolt radius that is not imposed either by the field equations or by the regularity of the geometry. Indeed, in the $k = 0, -1$ cases Misner string singularities are absent, and so there are no additional regularity conditions that can be imposed. However for the $k = 1$ spacetimes, regularity demands that these string singularities be removed. This imposes the additional requirement that σ be an integer. For all values of k, insertion of eq. (13) into the expression for the entropy yields an expression that is not proportional to the area of the bolt unless $N = 0$.

If $k \neq 1$, the parameters N, ℓ may take arbitrary values. For $N > 0$, we must take $r_b = r_{b+}$ (the upper branch) while for negative N we must take $r_b = r_{b-}$ (the lower branch). In this latter case the condition $r_b > N$ implies that upper limit $|\sigma| < (D+1)/2$. If $k = 1$ then the situation is more complicated. For $\sigma > (D+1)/2$ there are no constraints on N. However if $\sigma < (D+1)/2$ then either $N^2 < \frac{\ell^2}{2D} \left[1 - \sqrt{1 - \frac{4\sigma^2}{(D+1)^2}} \right]$ or $N^2 > \frac{\ell^2}{2D} \left[1 + \sqrt{1 - \frac{4\sigma^2}{(D+1)^2}} \right]$.

An investigation of the entropy and specific heat of these solutions reveals a general picture that resembles the $k = 1$, $D = 3$ case. For small N, the $k = 0, -1$ upper branch solutions have positive entropy and specific heat, and so are thermally stable, whereas the lower branch solutions are always unstable, with different signs for these quantities. The upper branch quantities diverge in the high temperature limit while the lower branch quantities vanish in the same limit. To illustrate this, in Fig. 1, we plot the entropy and specific heat per unit volume as function of N for the six-dimensional $k = 0, -1$ bolt solutions with $\ell = 1, |\sigma| = 1$.

It is also possible to consider situations in which $r_+ = N$. These are the NUT solutions noted above; the fixed point sets for the class of metrics that we are considering are 0-dimensional. For $k = -1$, we can prove that no such solutions exist, generalizing the results found in ref. [19] for $D = 3$. For $k = 0$ we find that for any D the function $F(r)$ has a double zero at $r_+ = N$. This implies that the $k = 0$ NUT solutions correspond to a background of arbitrary temperature since

the Euclidean time τ can be identified with arbitrary period.

We can write simple expressions for the parameter M, action and entropy of $k = 0, 1$ nut solutions

$$
\begin{aligned}
M_{k,D+1}^N &= \frac{\sqrt{\pi}}{2\ell^2}(-1)^{(D-1)/2} N^{D-2} \left(N^2(D+1) - kl^2\right) \frac{\Gamma(\frac{D+1}{2})}{\Gamma(\frac{D}{2})}, \\
I_{k,D+1}^N &= \frac{V\beta(-1)^{(D-1)/2}}{16\sqrt{\pi}G\ell^2} N^{D-2}\left((D-1)N^2 - k\ell^2\right) \frac{\Gamma(\frac{D+1}{2})}{\Gamma(\frac{D}{2})}, \qquad (14)\\
S_{k,D+1}^N &= \frac{V\beta(-1)^{(D-1)/2}}{16\sqrt{\pi}Gl^2} N^{D-2}\left(D(D-1)N^2 - (D-2)k\ell^2\right) \frac{\Gamma(\frac{D+1}{2})}{\Gamma(\frac{D}{2})},
\end{aligned}
$$

where again $\beta = 2\pi (D+1) N/\sigma$ in order to satisfy the first law (7). For $k = 0$, σ is arbitrary and we find that, for any D, there is no region in parameter space for which the entropy and specific heat are both positive definite.

We close by commenting on our results. We have shown that the breakdown of the entropy/area relationship arises as a consequence of the first law of thermodynamics in NUT-charged spacetimes. It does not depend on removal of Misner string singularities. However the Lorentzian sections of NUT charged spacetimes typically have closed timelike curves (CTCs) and so one might expect that our results are a consequence of this feature. In fact, this is not the case. Consider the curve generated by the Killing vector $\partial/\partial\varphi$ (for simplicity, we take here $D = 3$; the discussion in the higher dimensional case is similar). We note that

$$
g_{\varphi\varphi} = 4f_k^2(\frac{\theta}{2}) \left(r^2 + n^2 - f_k^2(\frac{\theta}{2})(4n^2 F + k(r^2 + n^2))\right),
$$

and so for all $k = 1, 0$ metrics and those $k = -1$ metrics with $4n^2/\ell^2 > 1$, the curve $r = r_0$, $t =$const., $\theta = \theta_0 \neq 0$ becomes timelike for large enough values of (r_0, θ_0), corresponding to a CTC since φ is a periodic coordinate (this type of causality violation is familiar from the study of the Gödel spacetime [20]). However there are also regions of these spacetimes that are free of this type of CTC provided $k \neq 1$ (the $k = 1$ metrics necessarily have a periodic time coordinate t [10]). Specifically, for the $k = -1$ metrics with $4n^2/\ell^2 \leq 1$, the function $h(x^i) = t$ is a global time coordinate (i.e. $g^{ij}(\partial h/\partial x^i)(\partial h/\partial x^j) < 0$ everywhere). Since these metrics also experience a breakdown of the entropy/area relationship, the existence of CTCs in the Lorentzian section cannot be the origin of this phenomenon.

Since in the cases of interest in this paper the dual CFTs are poorly known (see, e.g., ref. [21]), it is impossible to discuss them in detail. The best one can do at this point is to compare the (toy example) of scalar field thermodynamics on a Euclideanized Gödel-type background with the results obtained on the gravity side [22]. Using eq. (5) we obtain in 3 dimensions a *finite*, covariantly conserved and manifestly traceless dual stress tensor

$$
< \tau_b^a > = \frac{Mm^2}{8\pi G}[3u^a u_b + \delta_b^a],
$$

where $u^a = \delta_t^a$. Similar results hold in higher (odd) dimensions. As expected [23, 24, 25], this situation is different from the confined causality violating spacetimes, where the energy momentum

tensor of a quantum field diverges at the Cauchy horizon — unlike Gödel-type spacetimes, these have geodesic CTCs that dominate quantum amplitudes in the semiclassical picture. Interestingly enough, using the zeta function regularization approach a *finite* Euclidean effective action for the $k = 0$ NUT in $D = 3$, 5-dimensions was obtained in ref. [22]. Furthermore, it has the same sign and *similar* form to the bulk action (up to a scaling factor $(N/\ell)^{D-1}$) . These results indicate that the boundary CFT is well-defined, and that for this case the issue of chronology protection cannot be settled at the level of semiclassical quantum gravity but instead requires a full quantum gravity theory.

The presence of magnetic-type mass (the NUT parameter N) introduces for $k = 1$ a "Dirac-string singularity" in the metric (but no curvature singularity). We have shown for all k that the presence of a NUT charge yields additional contributions to the entropy that cause a breakdown of the entropy/area relationship regardless of the presence/absence of either Misner strings or CTCs. Rather it is a very basic feature of all NUT-charged spacetimes that any putative quantum theory of gravity will have to take into account. By extending our solutions to the asymptotically dS case, we expect that similar results will hold there as well.

Acknowledgments

References

[1] J. D. Bekenstein, Phys. Rev. **D7**, 2333 (1973).

[2] S. W. Hawking, Comm. Math. Phys. **43**, 199 (1975); R. M. Wald, Phys. Rev. **D48**, R3427 (1993); M. Ferraris, M. Francaviglia, in *Mechanics, Analysis and Geometry: 200 Years after Lagrange*, ed. M. Francaviglia, Elsevier Science Publishers B.V. (1991); L. Fatibene, M. Ferraris, M. Francaviglia, J. Math. Phys. **38**, 3953 (1997); J. D. E. Creighton and R. B. Mann, Phys. Rev. **D52**, 4569 (1995).

[3] A. Strominger and C. Vafa, Phys. Lett. **B379**, 99 (1996); A. Ashtekar, J. Baez, A. Corichi and K. Krasnov, Phys. Rev. Lett. **80**, 904 (1998); S. Carlip, Phys.Rev.Lett. **82**, 2828 (1999).

[4] G. W. Gibbons and S. W. Hawking, Phys. Rev. **D15**, 2752 (1977).

[5] V. Balasubramanian and P. Kraus, Commun. Math. Phys. **208**, 413 (1999).

[6] S. W. Hawking and C. J. Hunter, Phys. Rev. **D59**, 044025 (1999); S.W. Hawking, and C.J. Hunter, and D. Page, Phys. Rev. **D59**, 044033 (1999).

[7] R. B. Mann, Found. Phys. **33**, 65 (2003).

[8] A. M. Awad and A. Chamblin, Class. Quant. Grav. **19**, 2051 (2002).

[9] R. B. Mann and C. Stelea, Class. Class. Quant. Grav. **21**, 2937 (2004).

[10] C. W. Misner, J. Math. Phys. **4**, 924 (1963); C. W. Misner, in *Relativity Theory and Astrophysics I: Relativity and Cosmology*, edited by J. Ehlers, Lectures in Applied Mathematics, Volume 8 (American Mathematical Society, Providence, RI, 1967), p. 160.

[11] R. Emparan, C. V. Johnson and R. C. Myers, Phys. Rev. **D60**, 104001 (1999).

[12] R. B. Mann, Phys. Rev. **D60**, 104047 (1999).

[13] For a discussion and list of references see M. Visser, gr-qc/0204022.

[14] J. P. Gauntlett, J. B. Gutowski, C. M. Hull, S. Pakis and H. S. Reall Class. Quant. Grav. **20**, 4587-4634 (2003)

[15] D. A. Konkowski, T. M. Helliwell and L. C. Shepley, Phys. Rev. **D31**, 1178 (1985); D. A. Konkowski and T. M. Helliwell, Phys. Rev. **D31**, 1195 (1985).

[16] P. Kraus, F. Larsen and R. Siebelink, Nucl. Phys. **B563**, 259 (1999).

[17] S. Das and R. B. Mann, JHEP **0008**, 033 (2000).

[18] R. Clarkson, L. Fatibene and R. B. Mann, Nucl. Phys. **B652**, 348 (2003).

[19] A. Chamblin, R. Emparan, C. V. Johnson and R. C. Myers, Phys. Rev. **D59**, 064010 (1999).

[20] K. Gödel, Rev. Mod. Phys. **21**, 447 (1949).

[21] I. R. Klebanov and A. M. Polyakov, Phys. Lett. **B550**, 213 (2002).

[22] D. Astefanesei, R.B. Mann and E. Radu, hep-th/0407110 .

[23] Li-Xin Li, Class. Quant. Grav. **13**, 2563 (1996).

[24] R. Biswas, E. K. Vakkuri, R. G. Leigh, S. Nowling and E.Sharpe, JHEP **0401**, 064 (2004).

[25] J. P. Gauntlett, J. B. Gutowski and N. V. Suryanarayana, Class. Quant. Grav. **21**, 5021 (2004).